International Journal of Theoretical Physics, Vol. 42, No. 6, June 2003 (© 2003)

The Self-Induced Approach to Decoherence in Cosmology

Mario Castagnino[1,3] and Olimpia Lombardi[2]

Received April 14, 2003

In this paper we will present the *self-induced approach* to decoherence, which does not require the interaction between the system and the environment: decoherence in closed quantum systems is possible. This fact has relevant consequences in cosmology, where the aim is to explain the emergence of classicality in the universe conceived as a closed (noninteracting) quantum system. In particular, we will show that the self-induced approach may be used for describing the evolution of a closed quantum universe, whose classical behavior arises as a result of decoherence.

KEY WORDS: decoherence; cosmology.

1. INTRODUCTION

During the last years, the theory of decoherence has become the new orthodoxy in the quantum physicists community. At present, decoherence is studied and tested in many areas such as atomic physics, quantum optics, condensed matter, etc. (see references in Paz and Zurek, 2000; Zurek, 2001). Following the initial proposal of Zeh (1970), the theory was systematized and developed in a great number of works. According to Zurek (1991, 1994), decoherence is a process resulting from the interaction between a quantum system and its environment; this process singles out a preferred set of states, usually called "pointer basis," that determines which observables will receive definite values. This means that decoherence leads to a sort of selection which precludes all except a small subset of the states in the Hilbert space from behaving in a classical manner. Arbitrary superpositions are dismissed, and the preferred states become the candidate to classical states: they correspond to the definite readings of the apparatus pointer in quantum measurements, as well to the points in the phase space of a classical dynamical

[1] CONICET – Instituto de Física de Rosario, Rosario, Argentina.
[2] CONICET – Universidad de Buenos Aires. Buenos Aires, Argentina.
[3] To whom correspondence should be addressed at CONICET – Instituto de Física de Rosario, Avenue Pellegrini 250, Rosario, Argentina; e-mail: castagni@iafe.uba.ar.

system. *Environment-induced-superselection* (*einselection*) is a consequence of decoherence.

In this paper we will present a new approach to decoherence, different from the main stream einselection approach. From this new perspective, which we will call *self-induced approach*, decoherence does not require the interaction between the system and the environment: decoherence in closed quantum systems is possible. This fact has relevant consequences in cosmology, where the aim is to explain the emergence of classicality in the universe conceived as a closed (noninteracting) quantum system. In particular, we will show that the self-induced approach may be used for describing the evolution of a closed quantum universe, whose classical behavior arises as a result of decoherence.

2. THE EINSELECTION APPROACH IN COSMOLOGY

According to Zurek, einselected states are distinguished by their stability in spite of the monitoring environment. In Paz and Zurek's words (Paz and Zurek, 2000), "the environment distills the classical essence of a quantum system." This means that, from the einselection view, the split of the universe into the degrees of freedom which are of direct interest to the observer (the system) and the remaining degrees of freedom (the environment) is absolutely essential for decoherence. Such a split is necessary, not only for explaining quantum measurement, but also for understanding que quantum origin of the classical world. In fact, the einselection approach considers the problem of the transition from quantum to classical as the core of the problem: quantum measurement is conceived as a particular case of the general phenomenon of the emergence of classicality. In addition, if classicality only emerges in open quantum systems, it must always be accompanied by other manifestations of openness, such as dissipation of energy into the environment. Zurek (2001) even considers that the prejudice which seriously delayed the solution of the problem of the transition from quantum to classical is itself rooted in the fact that the role of the openness of a quantum system in the emergence of classicality was ignored for a very long time.

In summary, decoherence explains the emergence of classicality, but only open systems can decohere. The question is: What about the universe as a whole? If, as Zurek himself admits, the universe is, by definition, a closed system, then it cannot decohere. How to explain, then, the classical behavior of stars, galaxies, and clusters? Zurek (1994) considers this possible criticism: if the universe as a whole is a single entity with no "outside" environment, any resolution involving its division into systems seems unacceptable. Zurek's answer to this objection is based on his particular conception about the nature of quantum mechanics: for him, the aim of the theory is to establish the relationships between formal results and observer's perceptions. And perception is an information-processing function carried out by a physical system, the brain. The brain is conceived as a massive, neural network-like computer very strongly coupled to its environment, and the environment plays the role of a commonly accessible internet-like database, which allows the observer to make copies of the records concerning the states of the system with no danger of altering it Zurek (1998). The stability of the correlations between the state of the observer's brain and the state of the environment on the one hand, and between the state of the environment and the state of the observed system on the other, is responsible for the perception of classicality.

This means that, for the einselection approach, the problem of the transition from quantum to classical amounts to the question "why we don't perceive superpositions?" Zurek (1998). In other words, the task is to explain, not the emergence of classicality, but *our perception of classicality.* But this position would hardly convince the cosmologist, who conceives the universe as a single-closed object with no other object to interact with. In the cosmological context, the wave function of the universe describes, not the system of everything except the observers' brains, but the universe as a whole. Nevertheless, cosmology tries to explain, with the universal wave function, the evolution of a closed quantum universe where the classical behavior described by general relativity emerges. If we take Zurek's position seriously, without the assumption of a division of the universe into individual systems the problem of the emergence of classicality has no solution.

At this point, it could be noted that the einselection approach has been applied to the cosmological level with interesting results. This is certainly true, but does not undermine the closed-universe objection. In the works where the einselection approach is used in cosmology, the general strategy consists in splitting the universe into some degrees of freedom which represent the "system" of interest, and the remaining degrees of freedom that are supposed to be nonaccessible and, therefore, play the role of an internal environment. For instance, in quantum field theory, it is usual to perform a decomposition on a scalar field ϕ, $\phi = \phi_{\mathrm{S}} + \phi_{\mathrm{E}}$ where ϕ_{S} denotes the system field and ϕ_{E} denotes the environment field; when it is known that the background field follows a simple classical behavior, the scalar field is decomposed according to $\phi = \phi_c + \phi_q$, where the background field ϕ_c plays the role of the system and the fluctuation field ϕ_q plays the role of the environment (see Calzetta *et al.*, 2001). This means that, strictly speaking, it is not the universe what decoheres, but a subsystem of the universe: we perceive a classical universe because there are unaccessible degrees of freedom that act as an environment.

These considerations allow us to point out the weakest spot of the einselection program. When this approach is applied to the universe (and, in general, to any system with internal environment), the space of the observables which will behave classically is assumed in advance: the distinction between the system's degrees of freedom and the environmental degrees of freedom is established in such a way that the system decoheres in some observables of that space. This means that the split of the whole must be decided case by case: there is not a general criterion for discriminating between system and environment. In fact, in the case of the decomposition of the scalar field ϕ previously mentioned, different criteria are used: sometimes the decomposition is performed on the basis of the length, mass, or momentum scales of the system and the environment, sometimes the system field is considered as containing the lower modes of ϕ and the environment as containing the higher modes (see Calzetta *et al.*, 2001; Zurek (1998) recognizes that this lack of a general criterion for deciding where to place the "cut" between system and environment is a serious difficulty of his proposal: "In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the "systems" which play such a crucial role in all the discussions of the emergent classicality. This issue was raised earlier, but the progress to date has been slow at best."

As we will see, the self-induced approach to decoherence overcomes these problems to the extent that it does not require the openness of the system of interest and its interaction with the environment.

3. THE SELF-INDUCED APPROACH TO DECOHERENCE

This approach relies on the general idea that the interplay between observables and states is a fundamental element of quantum mechanics (see Laura and Castagnino, 1998a). The departing point consists in adopting an algebra of observables $\mathcal{A}$ as the primitive element of the theory: quantum states are represented by linear functionals over $\mathcal{A}$. In the original formulation of the algebraic formalism, the algebra of observables is a C*-algebra. The GNS theorem (Gel'fand–Naimark–Segal) proves that the traditional Hilbert space formalism is a particular representation of this algebraic formalism; the algebra of observables is thereby given a concrete representation as a set of self-adjoint bounded operators on a separable Hilbert space. Nevertheless, it is well known that the C*-algebraic framework does not admit unbounded operators; therefore, it is necessary to move to a less restrictive framework in order to accommodate this kind of operators. The self-induced approach adopts a *nuclear algebra* (Treves, 1967) as the algebra of observables $\mathcal{A}$: its elements are nuclei or kernels, that is, two variables distributions that can be though of as generalized matrices (Castagnino and Ordoñez, manuscript submitted for publication). By means of a generalized version of the GNS theorem (Iguri and Castagnino, 1999) it can be proved that this nuclear formalism has a representation in a rigged Hilbert space: the appropriate rigging provides a mathematical rigorous foundation to unbounded operators (see Belanger and Thomas, 1990). In fact, the nuclear spectral theorem of Gel'fand and Maurin establishes that, under very general mathematical hypotheses (quite reasonable from a physical point of view), for every CSCO (complete set of commuting observables) of essentially self-adjoint unbounded operators, there is a rigged Hilbert space where such a CSCO can be given a generalized eigenvalue decomposition, meaning that a continum of generalized eigenvalues and eigenvectors may thereby be associated with it. To find the appropriate rigging, the nuclear algebra is used to generate two additional topologies: one of them corresponds to a nuclear space, which is the space of generalized observables V_{O}; the other corresponds to a nuclear space, which is the space of generalized observables V_{O}; the other corresponds to the dual of the space V_{O}, and this is the space V_{S} of states.

Following (Antoniou *et al.*, 1997; Laura and Castagnino, 1998a,b), we will symbolize an observable belonging to V_{O} by a round ket $|O)$ and a state belonging to V_{S} by a round bra $(\rho|$. The result of the action of the round bra $(\rho|$ on the round ket $|O)$ is the mean value of the observable $|O)$ in the state $(\rho|$:

$$\langle O \rangle_{\rho} = (\rho|O) \tag{1}$$

If the basis is discrete, $\langle O \rangle_{\rho}$ can be computed as usual, that is, as $Tr(\rho O)$. But if the basis is continuous, $Tr(\rho O)$ is not well defined; nevertheless, $(\rho|O)$ can always be rigorously defined since $(\rho|$ is a linear functional belonging to V_{S} acting onto an operator $|O)$ belonging to V_{O}.

To see how decoherence works from the new approach, let us consider the simplest case, a quantum system whose Hamiltonian has a continuous spectrum:

$$H\,|\,\omega\rangle = \omega\,|\,\omega\rangle \qquad \omega \in [0, \infty) \tag{2}$$

where ω and $|\omega\rangle$ are the generalized eigenvalues and eigenvectors of H respectively. The CSCO of this system is just $\{H\}$. A generic observable $|O)$ can be expressed in terms of the eigenbasis $\{|\omega\rangle\}\langle\omega'|$ as

$$|O) = \iint \hat{O}(\omega, \omega')\,|\,\omega\rangle\langle\omega'\,|\,d\omega\,d\omega' = \iint \hat{O}(\omega; \omega')\,|\,\omega; \omega')d\omega\,d\omega' \tag{3}$$

where $|\omega; \omega') = |\omega\rangle\langle\omega|$ and $\hat{O}(\omega, \omega')$ represents the coordinates of the kernel $|O)$. The Hamiltonian in the eigenbasis $\{|\omega; \omega')\}$ reads

$$H = \int \omega\,|\,\omega\rangle\langle\omega\,|\,d\omega = \iint \omega\delta(\omega - \omega')\,|\,\omega; \omega')d\omega\,d\omega' \tag{4}$$

Then, $\omega\delta(\omega - \omega')$ must be one of $\hat{O}(\omega, \omega')$, since H is one of the observables belonging to V_{O}. Moreover, all the observables which commute with H and share the eigenbasis $\{|\omega; \omega')\}$ must have the following from:

$$|O) = \iint O(\omega)\,|\,\omega\rangle\langle\omega\,|\,d\omega = \iint O(\omega)\delta(\omega - \omega')\,|\,\omega; \omega')d\omega\,d\omega' \tag{5}$$

where $O(\omega)$ supplies the values of the components of $|O)$ in the basis $\{|\omega; \omega')\}$. Therefore, $O(\omega)\delta(\omega - \omega')$ must be one of $\hat{O}(\omega, \omega')$. But, of course, we need also observables which do not commute with H and whose $\hat{O}(\omega, \omega')$ are different than $O(\omega)\delta(\omega - \omega')$; then, with no loss of physical generality we can postulate as a general case:

$$\hat{O}(\omega, \omega') = O(\omega)\delta(\omega - \omega') + O(\omega, \omega') \tag{6}$$

where $O(\omega, \omega')$ is a regular function whose precise mathematical properties are listed in Castagnino and Laura (2000a).[4] Therefore, a generic observable $|O)$ reads (see van Hove, 1995)

$$|O) = \int O(\omega)\,|\,\omega)d\omega + \iint O(\omega, \omega')\,|\,\omega; \omega')\,d\omega\,d\omega' \tag{7}$$

where $|\omega) = |\omega\rangle\langle\omega|$ and $|\omega; \omega') = |\omega\rangle\langle\omega'|$ are the generalized eigenvectors of the observable $|O)$. We will call the first term of the r.h.s of Eq. (7) O_S (the singular part of the observable $|O)$). and the second term of the r.h.s of Eq. (7) O_R (the regular part of the observable $|O)$)

The observables $|O)$ of the form (7) define what we will call *van Hove space* $V_O^{VH} \subset V_O$; $\{|\omega),\ |\,\omega; \omega')\}$ is the basis of V_O^{VH}. On the other hand, states are represented by linear functionals belonging to a space V_S^{VH}, which is the dual of V_O^{VH}; therefore, a generic state $(\rho|$ can be expressed as

$$(\rho| = \int \rho(\omega)(\omega\,|\,d\omega + \iint \rho(\omega, \omega')\,(\omega; \omega'\,|\,d\omega\,d\omega' \tag{8}$$

where $\rho(\omega, \omega')$ is a regular function, and $\rho(\omega)$ and $\rho(\omega, \omega')$ satisfy the properties $\rho \geq 0, (\rho\,|\,I) = 1$ (where I) is the identity operator) and those listed in Castagnino and Laura (2000a). $\{(\omega|, (\omega; \omega'|\}$, the basis of V_S^{VH}, is the cobasis of $\{|\omega),\ |\,\omega; \omega')\}$ defined by the following relations[5]:

$$(\omega\,|\,\omega) = \delta(\omega - \omega') \quad (\omega; \omega''\,|\,\omega'; \omega''') = \delta(\omega - \omega'')\delta(\omega' - \omega''') \quad (\omega\,|\,\omega'; \omega'') = 0 \tag{9}$$

Given the expressions (7) and (8) for $|O)$ $(\rho|$ respectively, decoherence follows in a straightforward way. According to the unitary von Neumann equation, the evolution of $(\rho|$ is given by

$$(\rho(t)| = \int \rho(\omega)\,(\omega\,|\,d\omega + \iint \rho(\omega, \omega')\,e^{-i(\omega-\omega')t}(\omega; \omega'\,|\,d\omega\,d\omega' \tag{10}$$

Therefore, the mean value of the observable $|O)$ in the state $(\rho(t)|$ reads

$$\langle O\rangle_{\rho()t} = (\rho(t)\,|\,O) = \int \rho(\omega)O(\omega)d\omega + \iint \rho(\omega, \omega')e^{-i(\omega-\omega')t}O(\omega; \omega')d\omega\,d\omega' \tag{11}$$

Since $\rho(\omega, \omega')$ and $O(\omega, \omega')$ are regular functions (see Laura and Castagnino, 1998a, for details), if we take the limit for $t \to \infty$, we can apply the Riemann–Lebesgue theorem, according to which the second term of the r.h.s. of the last

[4] Since any singular kernel can be approximated by a regular one, the $O(\omega)\,\delta(\omega - \omega') + O(\omega, \omega')$ are dense on the set of $\hat{O}(\omega, \omega')$. Therefore, we do not lose physical generality, in the sense that $O(\omega)\,\delta(\omega - \omega') + O(\omega, \omega')$ have all the required physical properties up to any order and, then, they are experimentally indistinguishable from the $\hat{O}(\omega, \omega')$.

[5] These are the generalization of the relations between the basis $\{|i\rangle\}$ and the cobasis $\{\langle j|\}$ in the discrete case: $\langle j\,|\,i\rangle = \delta_{ij}$.

equation vanishes. Therefore,

$$\lim_{t\to\infty}\langle O\rangle_{\rho(t)} = \lim_{t\to\infty}(\rho(t)\mid O) = \int \rho(\omega)O(\omega)\,d\omega \tag{12}$$

But this integral is equivalent to the mean value of the observable O in a new state $(\rho_*|$:

$$(\rho_*| = \int \rho(\omega)\,(\omega|d\omega \tag{13}$$

where the off-diagonal terms have vanished. Therefore, we obtain the weak limit,

$$\lim_{t\to\infty}\langle O\rangle_{\rho(t)} = \langle O\rangle_{\rho*} \tag{14}$$

The next step is to study the formalism under the Wigner transformation "symb." Everything behaves in the usual way for the regular parts of $|O)$ and $(\rho|$, since these parts satisfy the hypotheses of papers (Hillary *et al.*, 1984; Wigner, 1932). The problem consists in defining the Wigner transformation for the singular parts. The singular parts of observables and states read

$$O_S = \int O(\omega)\mid\omega)d\omega = O(H) \qquad \rho_S = \int \rho(\omega)(\omega\mid d\omega \tag{15}$$

Therefore O_S is a function of the Hamiltonian,

$$H = \int \omega\mid\omega)d\omega$$

Using the well-known properties of the Wigner integral, we have that

$$symb O_S = O_S^W(q,p) = O(H^W(q,p)) + 0\left(\frac{\hbar^2}{S^2}\right) \tag{16}$$

where S is the characteristic action of the system. Then, in the particular case where $O(\omega) = \delta(\omega-\omega')$ we have from Eq. (16)

$$symb\mid\omega'\rangle\langle\omega'\mid = \delta(H^W(q,p)-\omega') \tag{17}$$

where we have disregarded the $0(\frac{\hbar^2}{S^2})$ as we will always do below. Moreover, for regular functions,

$$(\rho\mid O) = (symb\rho\mid symb O) = \int \rho^W(q,p)O^W(q,p)dqdp \tag{18}$$

We will adopt the same equation for the singular parts. This will allow us to define $symb\rho_S$ as satisfying

$$(symb\rho_S\mid symb O_S) = (\rho_S\mid O_S) \tag{19}$$

In doing so we are repeating what we have done at the quantum level when we defined the functional $(\rho|$.

From Eq. (17) we know that

$$symb|\omega') = \delta(H^{\mathrm{W}}(q, p) - \omega') \tag{20}$$

It is clear that we cannot normalize this function with the variables and in the domain of integration of (18). In fact, using the canonical variable $-t$, the canonical variable conjugated to H, for any function like $symb \mid \omega', p') = \delta(H^{\mathrm{W}}(\phi) - \omega')$, which is a constant for t, the integral will turn out to be infinity. So these functions $f(H)$ are not classical densities since they do not belong to L_1, if defined integrating over the whole phase space, and they must be normalized in a different way. This fact is not surprising since they are singular functions. But let us observe that, in general,

$$O_{\mathrm{S}}(\phi) = symb \int_0^{\infty} O(\omega) \mid \omega)\, d\omega = \int_0^{\infty} O(\omega)\, \delta(H^{\mathrm{W}}(\phi) - \omega')\, d\omega \tag{21}$$

which is a function independent of $-t$, and can be normalized (if necessary) imposing the following conditions:

i. We integrate only over the momentum space H (i.e. not over $-t$), precisely,

$$||O_{\mathrm{S}}(\phi)|| = \int dH \int_0^{\infty} \mid O(\omega, p) \mid \delta(H - \omega')\, d\omega = \int \mid O(H) \mid dH \tag{22}$$

ii. We chose the regular function $O(\omega)$ in the space L_1 of the momentum, that is to say,

$$\int \mid O(\omega) \mid d\omega < \infty \tag{23}$$

So, we will normalize all the $f(H)$ (if necessary) in this way, and we will perform *all the integrations in the $\mathcal{L}_{\mathrm{S}}$ space* by this method. In particular we will use this way of integration when *defining functionals of $\mathcal{L}'_{\mathrm{S}}$*.

Then, to satisfy Eq. (9), necessarily,

$$\rho^{\mathrm{W}}_{\mathrm{S}\omega}(q, p) = symb(\omega' \mid\, = \delta(H^{\mathrm{W}}(q, p) - \omega') \tag{24}$$

a result already obtained in Castagnino and Laura (2000b) (Eqs. (34) and (35)) with different methods. So, $\rho^{\mathrm{W}}_{\mathrm{S}\omega}(q, p)$ corresponds to a density function extremely peaked over the classical trajectory defined by the conservation law $H^{\mathrm{W}}(q, p) = \omega'$.

From Eq. (15),

$$\begin{aligned}\rho^{\mathrm{W}}_{\mathrm{S}}(q, p) &= \int_0^{\infty} \rho(\omega)\rho^{\mathrm{W}}_{\mathrm{S}\omega}(q, p)\, d\omega = \int_0^{\infty} \rho(\omega)\, \delta(H^{\mathrm{W}}(q, p) - \omega')\, d\omega \\ &= \rho(H^{\mathrm{W}}(q, p))\end{aligned} \tag{25}$$

and it is a *constant of the motion*, as it was expected. Moreover, $\rho_S^W(q, p) \geq 0$ since $\rho_S(\omega) \geq 0$. This means that $\rho_S^W(q, p)$ is the statistical ensemble of the density functions $\rho_{S\omega}^W(q, p)$ extremely peaked over the classical trajectories defined by the conservation law $H^W(q, p) = \omega'$ and weighted by the probabilities $\rho(\omega)$. From Eq. (14) we know that only the singular part must be considered after decoherence. Thus, for $t \to \infty$ the classical density $\rho_S^W(q, p)$ is resolved as a set of classical trajectories.

At this point, we have defined the Wigner transformation both for the regular parts and for the singular parts of observables and states. As a result, the singular parts share the same usual properties with the regular parts, since we have postulated such properties to define the Wigner transformation of the singular parts. In fact, these usual properties follow from Eq. (16) (see Castagnino and Laura, 2000b, for details). For example, Eq. (12) is also valid when the Wigner transformations are involved,

$$\lim_{t\to\infty} (\rho^W \mid O^W) = \int \rho_*^W(H) O_S^W(H) dH \tag{26}$$

This means that any observable $|O)$ becomes $O_S^W(q, p)$ in the limit $t \to \infty$, and behaves in a classical way.

After this presentation of the formalism in the simplest case, let us study the general case. In general, we must consider a CSCO, $\{H, O_1, \ldots, O_n\}$, whose eigenvectors are $|\omega, o_1, \ldots, o_n\rangle$. In this case, $(\rho_*|$ will be diagonal in the variables ω, ω' but not in general in the remaining variables. Therefore, a further diagonalization of $(\rho_*|$ is necessary: as a result, a new set of eigenvectors $\{|\omega, p_1, \ldots, p_n\rangle\}$, corresponding to a new CSCO $\{H, P_1, \ldots, P_n\}$ emerges. This set defines the eigenbasis $\{|\omega, p_1, \ldots, p_n), \omega, p_1, \ldots, p_n; \omega', p_1', \ldots, p_n'\}$ of the van Hove space of observables V_O^{VH}, where

$$\begin{aligned} |\omega, p_1, \ldots, p_n) &= |\omega, p_1, \ldots, p_n\rangle\langle\omega, p_1, \ldots, p_n| \\ |\omega, p_1, \ldots, p_n; \omega', p_1', \ldots, p_n') &= |\omega, p_1, \ldots, p_n\rangle\langle\omega', p_1', \ldots, p_n'| \end{aligned} \tag{27}$$

$(\rho_*|$ will be completely diagonal in the cobasis of states $\{(\omega, p_1, \ldots, p_n|, (\omega, p_1, \ldots, p_n; \omega', p_1', \ldots, p_n'|\}$ corresponding to the new eigenbasis of V_O^{VH} (see Castagnino and Laura, 2000b, for details). And, most important, for this system Eq. (25) reads

$$\begin{aligned} \rho_S^W(q, p) &= \sum_p \int_0^\infty \rho(\omega) \rho_{S\omega p_1,\ldots p_n}^W(q, p)\, d\omega \\ &= \sum_p \int_0^\infty \rho(\omega)\, \delta(H^W(q, p) - \omega')\, \delta\big(P_i^W(q, p) - p_i\big)\, d\omega \end{aligned} \tag{28}$$

where the density $\rho_{S\omega p_1,\ldots,p_n}^W$ corresponds to a density function extremely peaked over the classical trajectory defined by the conservation laws:

$$H^{W}(q, p) = \omega', \quad P_1^{W}(q, p) = p_1, \ \ldots \ P_n^{W}(q, p) = p_n \tag{29}$$

Therefore, for $t \to \infty$ the classical density $\rho_S^{W}(q, p)$ is resolved again as a set of classical trajectories.

As this presentation shows, decoherence does not require the interaction of the system of interest with the environment: *a single-closed quantum system can decohere*. The diagonalization of the density operator does not depend on the openness of the system but on the continuous spectrum of the system's Hamiltonian. This means that the problem of providing a general criterion for discriminating between system and environment vanishes in the self-induced approach. This fact leads to an additional advantage of the new way of conceiving decoherence. As we have seen, in many cases the einselection approach requires to introduce assumptions about the observables which will behave classically to decide where to place the boundary between system and environment. The new approach, on the contrary, provides a mathematically precise definition of the observables regarding to which the system will decohere. In fact, there are two kinds of such observables:

a) Observables that commute with the Hamiltonian, that are represented by the singular kernels $O(\omega)\,\delta(\omega - \omega')$
b) Observables that do not commute with the Hamiltonian, which are represented by the kernels $O(\omega)\,\delta(\omega - \omega') + O(\omega, \omega')$, where $O(\omega, \omega')$ is a regular function. In other words, these observables have a regular part $O(\omega, \omega')$ and a singular part $O(\omega)\,\delta(\omega - \omega')$ in the eigenbasis defined by the system's Hamiltonian.

This definition is completely general and does not require to introduce any prior assumption about the classical behavior of certain observables.

When the phenomenon of decoherence is viewed from this new perspective, it does not need to be conceived as "a justification for the persistent impression of reality" (Paz and Zurek, 2000). Classicality is not a perceptual result of the correlations between the observed system and the observer's brain though the environment: the emergence of classicality is a consequence of the own dynamics of a closed quantum system. In other words, from the self-induced approach, decoherence is a relevant element for explaining the *emergence* of classicality, not our *perception* of classicality.

4. DECOHERENCE IN A CLOSED UNIVERSE

If the transition from quantum to classical does not require the split of the universe into subsystems as a necessary condition, then decoherence can take part in the account of how the universe as a whole behaves classically. In this section we will apply the self-induced approach to a simple quantum-cosmological model to show how classicality arises in this case.

4.1. The Model

Let us consider the flat Roberson–Walker universe (Castagnino, 1998; Castagnino *et al.*, 1995; Castagnino and Lombardo, 1996; Paz and Sinha, 1991) with a metric,

$$ds^2 = a^2(\eta)(d\eta^2 - dx^2 - dy^2 - dz^2) \tag{30}$$

where η is the conformal time and a the scale of the universe. Let us consider a free neutral scalar field Φ and let us couple this field with the metric, with a conformal coupling ($\xi = \frac{1}{6}$). The total action reads $S = S_g + S_f + S_i$, and the gravitational action is

$$S_g = M^2 \int d\eta \left[-\frac{1}{2}\dot{a}^2 - V(a) \right] \tag{31}$$

where M is the Planck mass, $\dot{a} = da/d\eta$, and the potential V contains the cosmological constant term and, eventually, the contribution of some form of classical matter. We suppose that V has a bounded support $0 \leq a \leq a_1$. We expand the field $\mathbf{\Phi}$ as

$$\mathbf{\Phi}(\eta, \mathbf{x}) = \int_{-\infty}^{+\infty} f_{\mathbf{k}}(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} d\mathbf{k} \tag{32}$$

where the components of $\mathbf{k} \in \mathbb{R}^3$ are three continuous variables.

The Wheeler–De Witt equation for this model reads

$$H\Psi(a, \mathbf{\Phi}) = (h_g + h_f + h_i)\Psi(a, \mathbf{\Phi}) = 0 \tag{33}$$

where

$$\begin{aligned} h_g &= \frac{1}{2M^2}\partial_a^2 + M^2 V(a) \\ h_f &= -\frac{1}{2}\int \left(\partial_{\mathbf{k}}^2 - k^2 f_{\mathbf{k}}^2\right) d\mathbf{k} \\ h_i &= \frac{1}{2} m^2 a^2 \int f_{\mathbf{k}}^2 d\mathbf{k} \end{aligned} \tag{34}$$

with m the mass of the scalar field, $\mathbf{k}/a$ the linear momentum of the field, and $\partial_{\mathbf{k}} = \partial/\partial f_{\mathbf{k}}$.

We can now go to the semiclassical regime, using the WKB method (Hartle, 1995), writing $\Psi(a, \mathbf{\Phi})$ as

$$\Psi(a, \mathbf{\Phi}) = \exp[iM^2 S(a)]\chi(a, \mathbf{\Phi}) \tag{35}$$

and expanding S and χ as

$$S = S_0 + M^{-1}S_1 + \cdots, \qquad \chi = \chi_0 + M^{-1}\chi_1 + \cdots \tag{36}$$

To satisfy Eq. (33) at the order M^2, the principal Jacobi function, $S(a)$, must satisfy the Hamilton–Jacobi equation:

$$\left(\frac{dS}{da}\right)^2 = 2V(a) \tag{37}$$

We can now define the (semi)classical time as a parameter $\eta = \eta(a)$ such that

$$\frac{d}{d\eta} = \frac{dS}{da}\frac{d}{da} = \pm\sqrt{2V(a)}\frac{d}{da} \tag{38}$$

The solution of this equation is $a = \pm F(\eta, C)$, where C is an arbitrary integration constant. Different values of this constant and of the $\pm$ sign give different classical solutions for the geometry.

Then, in the next order of the WKB expansion, χ satisfies a Schrödinger equation that reads

$$i\frac{d\chi}{d\eta} = h(\eta)\chi \tag{39}$$

where

$$h(\eta) = h_f + h_i(a) \tag{40}$$

precisely

$$h(\eta) = -\frac{1}{2}\int\left[-\frac{\partial^2}{\partial f_{\mathbf{k}}^2} + \Omega_{\mathbf{k}}^2(a)\, f_{\mathbf{k}}^2\right]d\mathbf{k} \tag{41}$$

where

$$\Omega_{\mathbf{k}}^2(a) = \Omega_{\varpi}^2(a) = m^2a^2 + k^2 = m^2a^2 + \varpi \tag{42}$$

where $\varpi = k^2$ and $|k = |\mathbf{k}|$. So the time dependence of the Hamiltonian comes from the function $a = a(\eta)$.

Let us now consider a scale of the universe such that $a_{\text{out}} \gg a_1$. In this region the geometry is almost constant. Therefore, we have an adiabatic final vacuum $|0\rangle$ and adiabatic creation and annihilation operators $a_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}$. Then $h = h(a_{\text{out}})$ reads

$$h = \int \Omega_{\varpi} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} d\mathbf{k} \tag{43}$$

We can now consider the Fock space and a basis of vectors,

$$|\mathbf{k}_1, \mathbf{k}_2, \ldots, k_n\rangle \cong |\{k\}\rangle = a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger \ldots a_{\mathbf{k}_n}^\dagger \ldots |O\rangle \tag{44}$$

where we have called $\{k\}$ the set $\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_n$, where eventually n goes to infinity. The vectors of this basis are eigenvectors of h:

$$h|\{k\}\rangle = \omega|\{k\}\rangle \tag{45}$$

where

$$\omega = \sum_{\mathbf{k}\in\{\mathbf{k}\}} \Omega_{\varpi} = \sum_{\mathbf{k}\in\{\mathbf{k}\}} \left(m^2 a_{\text{out}}^2 + \varpi\right)^{\frac{1}{2}} \tag{46}$$

We can now use this energy to label the eigenvector as

$$|\{k\}\rangle = |\omega, [\mathbf{k}]\rangle \tag{47}$$

where $[\mathbf{k}]$ is the remaining set of labels necessary to define the vector unambiguously. $\{|\omega, [\mathbf{k}]\rangle\}$ is obviously an orthonormal basis, so Eq. (43) reads

$$h = \int \omega|\omega, [\mathbf{k}]\rangle\langle\omega, [\mathbf{k}]|\, d\omega\, d[\mathbf{k}] \tag{48}$$

4.2. Decoherence in Energy

In this case, a generic observable $|O) \in V_{\text{O}}^{\text{VH}}$ reads (see Eq. (7))

$$|O) = \int O(\omega, [\mathbf{k}], [\mathbf{k}'])|\omega, [\mathbf{k}]; [\mathbf{k}'])\, d\omega\, d[\mathbf{k}]\, d[\mathbf{k}'] \\ + \iint O(\omega, [\mathbf{k}], \omega', [\mathbf{k}'])|\omega, [\mathbf{k}]; \omega', [\mathbf{k}'])\, d\omega\, d[\mathbf{k}]\, d\omega' d[\mathbf{k}'] \tag{49}$$

where

$$|\omega, [\mathbf{k}]; [\mathbf{k}']) = |\omega, [\mathbf{k}]\rangle\langle\omega, [\mathbf{k}']|) \qquad |\omega, [\mathbf{k}]; \omega', [\mathbf{k}']) = |\omega, [\mathbf{k}]\rangle\langle\omega', [\mathbf{k}']|$$

and a generic state $(\rho| \in V_{\text{S}}^{\text{VH}}$ can be expressed as (see Eq. (8)):

$$|\rho) = \int \rho(\omega, [\mathbf{k}], [\mathbf{k}'])(\omega, [\mathbf{k}]; [\mathbf{k}']|\, d\omega\, d[\mathbf{k}]\, d[\mathbf{k}'] \\ + \iint \rho(\omega, [\mathbf{k}], \omega', [\mathbf{k}'])\, (\omega, [\mathbf{k}]; \omega', [\mathbf{k}']|\, d\omega\, d[\mathbf{k}]\, d\omega'\, d[\mathbf{k}'] \tag{50}$$

where $\{(\omega, [\mathbf{k}]; [\mathbf{k}']|, (\omega, [\mathbf{k}]; \omega', [\mathbf{k}']|\}$, the basis of V_{S}^{VH}, is the cobasis of $\{|\omega, [\mathbf{k}]; [\mathbf{k}']), |\omega, [\mathbf{k}]; \omega', [\mathbf{k}'])\}$. Then, the mean value of the observable $|O)$ in the state $(\rho(t)|$ reads

$$\langle O\rangle_{\rho(t)} = (\rho(t)|O) = \int \rho(\omega, [\mathbf{k}]; [\mathbf{k}'])O(\omega, [\mathbf{k}]; [\mathbf{k}'])\, d\omega\, d[\mathbf{k}]\, d[\mathbf{k}'] \tag{51}$$

$$+ \iint \rho(\omega, [\mathbf{k}], \omega', [\mathbf{k}'])e^{-i(\omega-\omega')t} \\ \times O(\omega, [\mathbf{k}], \omega', [\mathbf{k}'])\, d\omega\, d[\mathbf{k}]\, d\omega'\, d[\mathbf{k}'] \tag{52}$$

Taking the limit for $t \rightarrow \infty$ and applying the Riemann–Lebesgue theorem, we obtain

$$\lim_{t\rightarrow\infty} \langle O \rangle_{\rho(t)} = \lim_{t\rightarrow\infty} (\rho(t)|O) = \int \rho(\omega, [\mathbf{k}]; [\mathbf{k}'])O(\omega, [\mathbf{k}]; [\mathbf{k}'])\, d\omega\, d[\mathbf{k}]\, d[\mathbf{k}'] \tag{53}$$

And this integral is equivalent to the mean value of the observable $|O)$ in a new state $(\rho_*|$,

$$(\rho_*| = \int \rho(\omega, [\mathbf{k}]; [\mathbf{k}'])\, (\omega, [\mathbf{k}]; [\mathbf{k}'] \,|\, d\omega\, d[\mathbf{k}]\, d[\mathbf{k}'] \tag{54}$$

This new state $(\rho_*|$ is the equilibrium time-asymptotic state, which is diagonal to the variables ω, ω' as decoherence in energy requires.

4.3. Decoherence in the Remaining Dynamical Variables

In this case, $(\rho_*|$ is diagonal in the variables ω, ω' but not in the remaining variables. This means that a further diagonilization is necessary.

Let us observe that, if we use polar coordinates for $\mathbf{k}$, Eq. (32) reads

$$\mathbf{\Phi}(x, n) = \int \sum_{lm} \phi_{klm} dk \tag{55}$$

where

$$\phi_{klm} = f_{k,l}(\eta, r) Y_m^l(\theta, \varphi) \tag{56}$$

where k is a continuous variable, $l = 0, 1, \ldots$; $m = -l, \ldots, l$; and Y_m^l are spherical harmonic functions. So the indices k, l, m contained in the symbol $\mathbf{k}$ are partially discrete and partially continuous.

As $(\rho_*^\dagger| = (\rho_*|$, then $\rho^*(\omega, [\mathbf{k}], [\mathbf{k}']) = \rho(\omega, [\mathbf{k}], [\mathbf{k}'])$ and, therefore, there exists a set of vectors $\{|\omega, [\mathbf{l}]\rangle\}$ such that

$$\int \rho(\omega, [\mathbf{k}], [\mathbf{k}'])|\omega, [\mathbf{l}]\rangle_{[\mathbf{k}']} d[\mathbf{k}'] = \rho(\omega, [\mathbf{l}])|\omega, [\mathbf{l}]\rangle_{[\mathbf{k}]} \tag{57}$$

namely, $\{|\omega, [\mathbf{l}]\rangle\}$ is the eigenbasis of the operator $\rho(\omega, [\mathbf{k}], [\mathbf{k}'])$. Then $\rho(\omega, [\mathbf{l}])$ can be considered as an ordinary diagonal matrix in the discrete indices l and m, and a generalized diagonal matrix in the continuous index k.[6] Under the diagonalization

[6] For example, we can deal with this generalized matrix by rigging the space $V_{\mathrm{S}}^{\mathrm{VH}}$ and using the Gel'fand-Maurin theorem (Parravicini *et al.*, 1980); this procedure allows us to define a generalized state eigenbasis for systems with continuous spectrum. It has been used to diagonalize Hamiltonians with continuous spectra the literature (Bohm, 1986; Castagnino *et al.*, 1996; Castagnino and Laura, 1997), etc.

process, Eq. (54) is written as

$$(\rho_*| = \int U^{\dagger[\mathbf{l}]}_{[\mathbf{k}]} \rho(\omega, [\mathbf{l}], [\mathbf{l}']) U^{[\mathbf{l}']}_{[\mathbf{k}']} U^{\dagger[\mathbf{l}'']}_{[\mathbf{k}']} (\omega, [\mathbf{l}''], [\mathbf{l}'''] \,|\, U^{[\mathbf{l}''']}_{[\mathbf{k}]} \times d\omega\, d[\mathbf{k}]\, d[\mathbf{k}']\, d[\mathbf{l}]\, d[\mathbf{l}']\, d[\mathbf{l}'']\, d[\mathbf{l}'''] \tag{58}$$

where $U^{\dagger[\mathbf{l}]}_{[\mathbf{k}]}$ is the unitary matrix used to perform the diagonalization and[7]

$$\rho(\omega, [\mathbf{l}], [\mathbf{l}']) = \rho(\omega, [\mathbf{l}])\delta([\mathbf{l}] - [\mathbf{l}']) \tag{59}$$

Since

$$\rho(\omega, [\mathbf{l}], [\mathbf{l}]) = \rho(\omega, [\mathbf{l}]) = \int U^{[\mathbf{k}]}_{[\mathbf{l}]} \rho(\omega, [\mathbf{k}], [\mathbf{k}']) U^{\dagger[\mathbf{k}']}_{[\mathbf{l}]}\, d[\mathbf{k}]\, d[\mathbf{k}'] \tag{60}$$

we can define

$$(\omega, [\mathbf{l}]| = (\omega, [\mathbf{l}], [\mathbf{l}]| = \int U^{[\mathbf{k}]}_{[\mathbf{l}]} (\omega, [\mathbf{k}], [\mathbf{k}']| U^{\dagger[\mathbf{k}']\dagger}_{[\mathbf{l}]}\, d[\mathbf{k}]\, d[\mathbf{k}'] \tag{61}$$

We can repeat the procedure with vectors $(\omega, \omega', [\mathbf{k}], [\mathbf{k}']|$ and obtain vectors $(\omega, \omega', [\mathbf{l}]|$. In this way we obtain a diagonalized cobasis $\{(\omega, [\mathbf{l}]|, (\omega, \omega', [\mathbf{l}]|\}$. So we can now write the equilibrium state as

$$(\rho_*| = \int \rho(\omega, [\mathbf{l}])(\omega, [\mathbf{l}] \,|\, d\omega\, d[\mathbf{l}] \tag{62}$$

Since vectors $(\omega, [\mathbf{l}]|$ can be considered as diagonal in all the variables, we have obtained decoherence in all the dynamical variables.

4.4. Emergence of Trajectories

Let us restore the notation $\{l\} = (\omega, [\mathbf{l}])$, $\{k\} = (\omega, [\mathbf{k}])$ as in Eq. (44), and let us consider the configuration kets $|\{x\}\rangle = |\eta, [\mathbf{x}]\rangle$. Since we are considering the period when $a \sim a_{\text{out}}$, the system with Hamiltonian (43) is just a set of infinite oscillators with constants $\Omega_{\mathbf{k}}(a_{\text{out}})$ that represents a scalar field with mass ma_{out}. Then, we are just dealing with a classical set of N particles, with coordinates $[\mathbf{x}]$ and momenta $[\mathbf{k}]$. Therefore, we can introduce the Wigner function corresponding to the generalized state $|\{l\})$,

$$\rho^{\mathrm{W}}_{\{l\}}([\mathbf{x}], [\mathbf{k}]) = \pi^{-4N} \int (\{l\} \,|\, \mathbf{x} + \lambda\rangle \langle \mathbf{x} - \lambda|)\, e^{2i[\lambda]\cdot[\mathbf{k}]}\, d^{4n}\lambda \tag{63}$$

Using the same reasoning that we have used to obtain Eq. (28) (see also Castagnino and Laura, 2000b) $\rho^{\mathrm{W}}_{\{l\}}([\mathbf{x}], [\mathbf{k}])$ reads

$$\rho^{\mathrm{W}}_{\{l\}}([\mathbf{x}], [\mathbf{k}]) = \prod_i \delta\big(L^{\mathrm{W}}_i([\mathbf{x}], [\mathbf{k}]) - l_i\big) \tag{64}$$

[7] $\delta([\mathbf{l}] - [\mathbf{l}]')$ is a Dirac delta for the continuous indices and a Kronecker delta for the discrete ones.

where $L_i^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$ is the classical observable obtained from L_i (that corresponds to indices $\mathbf{l}$) via the Wigner integral (considering $h = L_0$ and including 0 among the indices i). Now, with the new notation Eq. (62) reads

$$(\rho_*| = \int \rho_{\{l\}}(\{l\} \,|\, d\{l\} \tag{65}$$

Then, if we call

$$\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) = \pi^{-4N} \int (\rho_* | \mathbf{x} + \lambda\rangle \langle \mathbf{x} - \lambda |)\, e^{2i[\lambda]\cdot[\mathbf{k}]}\, d^{4n}\lambda \tag{66}$$

we obtain

$$\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) = \rho_*^{\mathrm{W}}\big(L_0^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]),\, L_1^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]), \ldots\big) \tag{67}$$

So finally,

$$\begin{aligned} \rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) &\sim \int \rho_{\{l\}} \rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])\, \delta(\{L^{\mathrm{W}}\} - \{l\}\, d\{l\} \\ &= \int \rho_{\{l\}} | \prod_i \delta\big(L_i^{\mathrm{W}} - l_i\big)\, d\{l\} \end{aligned} \tag{68}$$

The last equation can be interpreted as follows:

i. $\delta(\{L^{\mathrm{W}}\} - \{l\})$ is a classical density function, strongly peaked at certain values of the constants of motion $\{l\}$, corresponding to a set of trajectories, where the momenta are equal to the eigenvalues $l_i (i = 0, 1, 2, \ldots)$.

ii. $\rho_{\{l\}}$ is the probability to be in one of these sets of trajectories labelled by $\{l\}$. Precisely, if some initial density matrix is given, from Eq. (65) it is evident that its diagonal terms $p_{\{l\}}$ are the probabilities to find the density function $\delta(\{L^{\mathrm{W}}\} - \{l\})$ in the corresponding classical equilibrium density function $\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$ namely, the probabilities of the trajectories labelled by $\{l\} = (\omega, [\mathbf{l}])$.

iii. Let $\mathbf{a}$ be the coordinate classically conjugated to $\mathbf{l}$ and let $\mathbf{a}_0$ be the coordinate $\mathbf{a}$ at time $\eta = 0$,[8] then we obtain the classical trajectories:

$$\mathbf{a}_i = \mathbf{l}_i \eta + \mathbf{a}_{0i} \tag{69}$$

iv. Let us now call $\rho_{\{l\}} = p_{\{l\}}[\mathbf{a}_0]$. Actually, $p_{\{l\}[\mathbf{a}_0]}$ is not a function of $\mathbf{a}_0$; it is simply a constant in $\mathbf{a}_0$, since $\mathbf{a}_0$ is only an arbitrary point and our model is spatially homogenous. Then we can write

$$p_{\{l\}[\mathbf{a}_0]} = \int p_{\{l\}[\mathbf{a}_0]} \prod_{i=1}^{n} \delta(\mathbf{a}_i - \mathbf{a}_{0i})\, d[\mathbf{a}_0] \tag{70}$$

[8] We could also add a nonrelevant equation, something like $t = \omega\eta + t_0$, which would define a choice of our clock's time.

In this way we have changed the role of $\mathbf{a}_0$: it was a fixed (but arbitrary) point, and now it is a variable that moves all over the space. Then Eq. (68) reads

$$\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) \sim \int p_{\{l\}[\mathbf{a}_0]} \prod_i^n \delta(L_i^{\mathrm{W}} - l_i) \prod_{j=1}^n \delta(\mathbf{a}_j - \mathbf{a}_{0j}) d[\mathbf{a}_0]\, d\{l\} \tag{71}$$

So, if we call

$$\rho_{\{l\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) = \prod_{i=0}^n \delta(L_i^{\mathrm{W}} - l_i) \prod_{j=1}^n \delta(\mathbf{a}_j - \mathbf{a}_{0j}) \tag{72}$$

we have

$$\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) \sim \int p_{\{l\}[\mathbf{a}_0]} \rho_{\{l\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])\, d[\mathbf{a}_0]\, d\{l\} \tag{73}$$

From Eq. (72) we see that $\rho_{\{l\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) \neq 0$ only in a narrow strip around the classical trajectory (69) defined by the momenta $\{l\}$ and passing through the point $[\mathbf{a}_0]$ (actually the density function is as peaked as it is allowed by the uncertainty principle; its width is essentially a $0(\frac{\hbar}{S})$, since the δ functions of all the equations become Dirac's deltas only when $\hbar \to 0$). Therefore, Eq. (73) describes the classical behavior of our model of universe.[9]

Let us sum up the main steps of our argument. When $\eta \to \infty$, the quantum density ρ becomes a diagonal density matrix ρ_*. The corresponding classical distribution $\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$ can be expanded as a sum of classical trajectories density functions $\rho_{\{l\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$, each one weighted by its corresponding probability $p_{\{l\}[\mathbf{a}_0]}$. Going back to Eq. (73), $\rho_{\{l\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}]) = \rho_{\{\mathbf{l}_1, \mathbf{l}_2, \ldots \mathbf{l}_n\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$ is the density corresponding to the set of n points (let us say, galaxies), each one of them moving over a trajectory defined by Eq. (69), where eventually n goes to infinity. So, as the limit of our quantum model we have obtained a classical statistical mechanical model, and the classical realm appears.

In summary, we have proved that the density operator is translated into a classical density, via a Wigner function, and it is decomposed as a sum of densities peaked around all possible classical trajectories, each one of these densities

[9] In this section, as in Section 2.2, we have faced the following problem: $\rho_*^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$ is a a constant that we want to decompose in functions $\rho_{\{l\}[\mathbf{a}_0]}^{\mathrm{W}}([\mathbf{x}], [\mathbf{k}])$ which are different from zero only around the trajectory (69) and therefore are variables in $\mathbf{a}$. Then, essentially we use the fact that if $f(x, y) = g(y)$ is a constant function in x, we can decompose it as

$$g(y) = \int g(y)\, \delta(x - x_0)\, dx_0$$

namely, the densities $\delta(x - x_0)$ are peaked in the trajectories $x = x_0 = \text{const.}$, $y = \text{var.}$ and, therefore, are functions of x. This trajectories play the role of those of Eq. (70).

As all the physics, including the correlations, is already contained in Eq. (68), the reader may just consider the final part of this section, from Eq. (70) to Eq. (73) a didactical presentation.

weighted by their own probability. Therefore, our quantum density operator behaves in its classical limit as a statistical distribution among a set of classical trajectories. Similar results are obtained in papers by Halliwell and Zouppas (1995) and Polarsky and Starobinsky (1996).

5. OPERATORS AND FIELDS

Up to this point, all our reasoning was made in the context of the Schrödinger picture, as it is usual in quantum mechanics. But in quantum field theory, on the contrary, the Heisenberg picture is the usual scenario. However, it is quite easy to reformulate the argument in the new picture, since our main equations are (14) and (26) for the general case, and (53) and its corresponding classical version for the cosmological case. In both cases only the singular parts O_S and $\rho_S = \rho_*$ appear, and both functions are time independent. Then, whereas in the Schrödinger picture we have that $O = O(t_0)$ is time constant and $\rho(t) \to \rho_* = \rho_S$ is time constant in the limit, in the Heisenberg picture $\rho = \rho(t_0)$ is time constant and $O(t) \to O_* = O_S$ is time-constant in the limit; nevertheless, both results coincide since when $t = \infty$, $\rho_* = \rho_S$, and $O_* = O_S$. Then, if we consider the Heisenberg picture, when $t \to \infty$ the operators $f_{\mathbf{k}}(\eta)$ of Eq. (32) become singular and also equal to the constant $f_{\mathbf{k}}$; therefore, the corresponding $f_{\mathbf{k}}^{\mathrm{W}}$ are constant and classical. The same can be said for the field $\mathbf{\Phi}(\eta, \mathbf{x})$ of Eq. (32), that becomes a time constant $\mathbf{\Phi}(\mathbf{x})$.

With this procedure, in certain sense we have lost the dynamics of the field. However, it can be recovered if we compute a "master equation" from any asymptotic expansion of the Riemann–Lebesgue theorem or if we make an analytic continuation in the Liouville complex energy plane as in paper by Arbó *et al.* (2000). Such a master equation coincides with the usual one and also with the Lindblad approach.

6. CONCLUSION

In this paper our aim was to rigorously present the self-induced approach to decoherence, according to which the phenomenon of decoherence is the result of the own dynamics of a closed system governed by a Hamiltonian with continuous spectrum. From this approach, the interaction between the system and the environment is not needed and, therefore, a single-closed quantum system can decohere. This feature makes this approach particularly appropriate for addressing problems in quantum cosmology, since the universe is, by definition, a closed quantum system with no environment to interact with. In this context, we have applied the self-induced approach to a quantum cosmological model, showing that decoherence in energy and in the remaining dynamical variables obtains with no reference to an environment and without assuming in advance which observables will behave classically.

REFERENCES

Antoniou, I., Suchanecki, Z., Laura, R., and Tasaki, S. (1997). *Physica Status Solidi A: Applied Research* 241, **737**.

Arbó, D., Castagnino, M., Gaioli, F., and Iguri, S. (2000). *Physica Status Solidi A: Applied Research* **277**, 469. [quant-ph/0000541].

Belanger, A. and Thomas, G. F. (1990). *Canadian Journal of Mathematics* **42**, 410.

Bohm, A. (1986). *Quantum Mechanics, Foundations and Applications*, Springer-Verlag, Berlin, Germany.

Calzetta, E. A., Hu, B. L., and Mazzitelli, F. D. (2001). *Physics Reports* **352**, 459.

Castagnino, M. (1998). *Physical Review D: Particles and Fields* **57**, 750.

Castagnino, M., Gaioli, F., and Gunzig, E. (1996). *Foundations Cos. Physics* **16**, 221.

Castagnino, M., Gunzig, E., and Lombardo, F. (1995). *General Relativity and Gravitation* **27**, 257.

Castagnino, M. and Laura, R. (1997). *Physical Review A* **56**, 108.

Castagnino, M. and Laura, R. (2000a). *International Journal of Theoretical Physics* **39**, 1737.

Castagnino, M. and Laura, R. (2000b). *Physical Review A* **62**, #022107.

Castagnino, M. and Lombardo, F. (1996). *General Relativity and Gravitation* **28**, 263.

Castagnino, M. and Ordoñez, A. (manuscript submitted for publication). *Journal of Physics A: Mathematical and General*.

Halliwell, J. and Zouppas, A. (1995). *Physical Review D: Particles and Fields* **52**, 7294.

Hartle, J. (1985). In High energy physics, 1985. *Proceedings of the Yale Summer School*, N. J. Bowik and F. Gursey, eds., World Scientific, Singapore.

Hillary, M., O'Connell, R., Scaully, M., and Wigner, E. (1984). *Physics Reports* **106**, 123.

Iguri, S. and Castagnino, M. (1999). *International Journal of Theoretical Physics* **38**, 143.

Laura, R. and Castagnino, M. (1998a). *Physical Review A* **57**, 4140.

Laura, R. and Castagnino, M. (1998b). *Physical Review E* **57**, 3948.

Parravicini, G., Gorini, V. and Sudarshan, E. C. G. (1980). *Journal of Mathematial Physics* **21**, 2208.

Paz, J. P. and Sinha, S. (1991). *Physical Review D: Particles and Fields* **44**, 1038.

Paz, J. P. and Zurek, W. H. (2000). *LANL. Preprint* quant-ph/0010011.

Polarky, D. and Starobinsky, A. A. (1996). *Classical and Quantum Gravity* **13**, 377.

Treves, A. (1967). *Topological Vector Spaces, Distributions and Kernels*, Academic Press, New York.

van Hove, L. (1955). *Physica* **21**, 901.

Wigner, E. P. (1932). *Physical Review* **40**, 749.

Zeh, D. (1970). *Foundations of Physics* **1**, 69.

Zurek, W. H. (1991). *Physics Today*, **44**, 36.

Zurek, W. H. (1994). In *Physical Origins of Time Asymmetry*, J. J. Halliwell, J. Pérez-Mercader, and W. H. Zurek, eds., Cambridge University Press, Cambridge, UK.

Zurek, W. H. (1998). *LANL. Preprint* quant-ph/9805065.

Zurek, W. H. (2001). *LANL. Preprint* quant-ph/0105127.